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PURE MATHEMATICS 10b–20b

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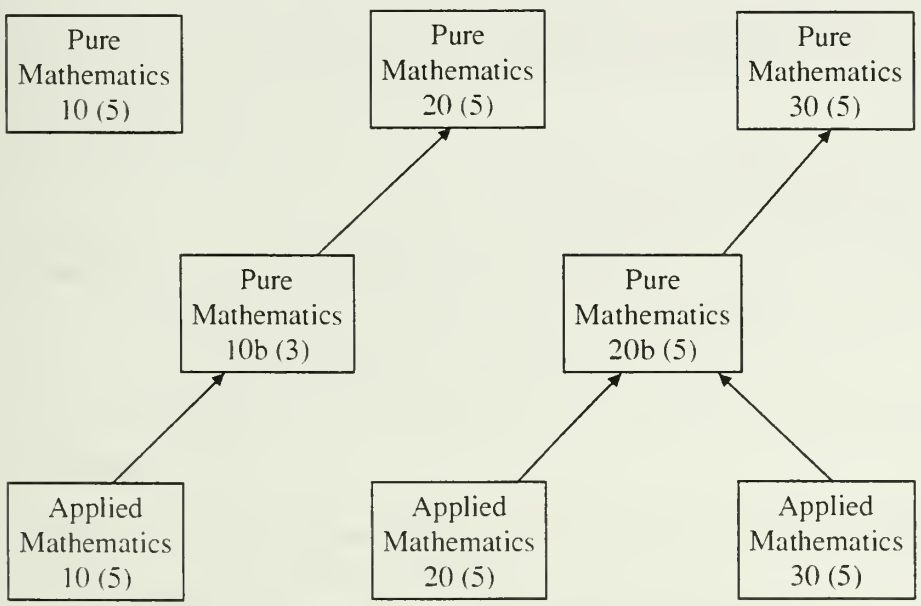
PURE MATHEMATICS 10b–20b

A. RATIONALE AND PHILOSOPHY

Students, for a variety of reasons, may want to transfer from the applied mathematics course sequence to the pure mathematics course sequence. In this case, students must satisfy the prerequisites that cover the outcomes exclusive to pure mathematics. The outcomes common to pure mathematics and applied mathematics need not be repeated. For those students who want to transfer after meeting the acceptable standard in Applied Mathematics 10, the prerequisite is Pure Mathematics 10b, a 3-credit course. For those students who want to transfer after meeting the acceptable standard in Applied Mathematics 20 or Applied Mathematics 30, the prerequisite is Pure Mathematics 20b, a 5-credit course. The two bridging courses are shown in the diagram below.


Pure Mathematics 10 (5)
Pure Mathematics 20 (5)
Pure Mathematics 30 (5)
Pure Mathematics 10b (3)
Pure Mathematics 20b (5)
Applied Mathematics 10 (5)
Applied Mathematics 20 (5)
Applied Mathematics 30 (5)

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B. LEARNER OUTCOMES

The general outcomes for the Pure Mathematics 10b–20b program are shown first. For each of the two courses, the learner outcomes follow the format of general outcome, specific outcomes and illustrative examples. The illustrative examples are for discretionary use only. The outcomes are arranged in order of strand and substrand, following the order—Number, Patterns and Relations, Shape and Space, Statistics and Probability—established in the Grade 10 to Grade 12 common curriculum framework. This listing is not intended as a sequence for instruction, and the number of outcomes is not an indication of the relative times to be spent on each of the strands. All the specific outcomes have a **P** prefix, such as P1–7, and they refer to those outcomes that are exclusive to pure mathematics.

The learner outcomes listing for Pure Mathematics 10b includes all those outcomes that are exclusive to Pure Mathematics 10.

The learner outcomes listing for Pure Mathematics 20b includes all those outcomes that are exclusive to Pure Mathematics 10 and Pure Mathematics 20.

Outcomes that are exclusive to Pure Mathematics 10 are printed twice, once as Pure Mathematics 10b and once as part of Pure Mathematics 20b.

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GENERAL OUTCOMES—Number Strand

Substrand	Pure Mathematics 10b–20b
Number Operations *Students will:* • demonstrate an understanding of and proficiency with calculations • decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.	Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems. [Pure Mathematics 10b and 20b]

GENERAL OUTCOMES—Patterns and Relations Strand

Substrand	Pure Mathematics 10b–20b
Patterns *Students will:* • use patterns to describe the world and to solve problems.	Generate and analyze number patterns. [Pure Mathematics 10b and 20b] Apply the principles of mathematical reasoning to solve problems and to justify solutions. [Pure Mathematics 20b]
Variables and Equations *Students will:* • represent algebraic expressions in multiple ways.	Generalize operations on polynomials to include rational expressions. [Pure Mathematics 10b and 20b] Represent and analyze situations that involve expressions, equations and inequalities. [Pure Mathematics 20b]
Relations and Functions *Students will:* • use algebraic and graphical models to generalize patterns, make predictions and solve problems.	Examine the nature of relations with an emphasis on functions. [Pure Mathematics 20b] Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate. [Pure Mathematics 20b]

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GENERAL OUTCOMES—Shape and Space Strand

Substrand	Pure Mathematics 10b–20b
Measurement *Students will:* • describe and compare everyday phenomena, using either direct or indirect measurement.	Solve problems involving triangles, including those found in 3-D and 2-D applications. [Pure Mathematics 20b]
3-D Objects and 2-D Shapes *Students will:* • describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	Solve coordinate geometry problems involving lines and line segments, and justify the solutions. [Pure Mathematics 20b] Develop and apply the geometric properties of circles and polygons to solve problems. [Pure Mathematics 20b]

GENERAL OUTCOMES—Statistics and Probability Strand

Substrand	Pure Mathematics 10b–20b
Chance and Uncertainty *Students will:* • use experimental or theoretical probability to represent and solve problems involving uncertainty.	Make and analyze decisions, using expected gains and losses, based on the probabilities of simple events. [Pure Mathematics 10b and 20b]

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Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems. **Specific Outcomes** P1–1. Explain and apply the exponent laws for powers of numbers and for variables with rational exponents. [C, E]	1.1 Find the exact value of $\left(\frac{8}{27}\right)^{\left(-\frac{2}{3}\right)}$. 1.2 Write the number expression $7^{\left(\frac{2}{3}\right)}$, using radicals. 1.3 Simplify $\left(\sqrt[5]{x^3}\right)\left(\sqrt[3]{x^2}\right)$. 1.4 Show $\left(\sqrt[3]{-8}\right)x = -2x$. 1.5 Write an equivalent expression for $\sqrt[3]{\sqrt[2]{3x^5}}$, using exponents. 1.6 Prove that $\sqrt{2}$ is an irrational number. 1.7 The 5×5 geoboard shown in the diagram can be used to construct squares whose areas are whole numbers. The sides of the squares can be constructed by joining dots horizontally, vertically or diagonally. What whole number areas can be constructed? Justify your answers with appropriate drawings and calculations. . . . .

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Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P2–1. Perform operations on irrational numbers of monomial and binomial form, using exact values. [E]	1.1 Show that $\sqrt{2}+\sqrt{8}=3\sqrt{2}$. 1.2 Find an equivalent form of $\left(\frac{3}{\sqrt{5}-\sqrt{2}}\right)$ that has a whole number as its denominator. 1.3 Arrange the following in order from least to greatest: $7,\ 2\sqrt{13},\ 3\sqrt{6},\ 4\sqrt{5},\ 5\sqrt{2}$. Do not use decimal approximations. 1.4 Find the exact value of $\sqrt[3]{128}+4\left(\sqrt[3]{16}\right)$. 1.5 Find an equivalent form of $\left(3\sqrt{5}+4\sqrt{2}\right)\left(4\sqrt{5}-3\sqrt{2}\right)$. 1.6 An equilateral triangle is inscribed in a circle. If the area of the circle is 36π, find the exact area of the equilateral triangle.

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Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Generate and analyze number patterns. **Specific Outcomes**	
P2–2. Generate number patterns exhibiting arithmetic growth. [E, R]	2.1 The first modern Olympiad was held in 1896. Every four years after this date the summer Olympics were held. Given such a framework, reveal what should have been the next five summer Olympic years after 1896. Explain why this pattern was never achieved. 2.2 The output of a northern gold mine has remained constant at 2200 ounces per year. If, at the end of last year, the total output of the mine was 122 600 ounces of gold, what will be the total output at the end of this year? At the end of next year? 2.3 A salesperson receives a base salary of $12 000 per year, plus $100 for every unit sold. What is the salary, if 50 units are sold? 51 units? 52 units? 2.4 For the arithmetic sequence 16, 23, 30, 37, . . . , find the next three terms. 2.5 A pile of bricks is arranged in rows. The numbers of bricks in the rows form an arithmetic sequence. There are 45 bricks in the 5th row and 33 bricks in the 11th row. a) How many bricks are in the first row? b) Write the general term for the sequence. c) What is the maximum number of rows of bricks possible?
P2–3. Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems. [CN, PS, R, T]	3.1 For the arithmetic sequence 7, 11, 15, 19, . . . , find the 29th term. 3.2 Find the sum of the arithmetic series $3 + 7 + 11 + \ldots + 483$. 3.3 Mary's annual salary is on a range from $26 785 in the first year to $34 825 in the seventh year. a) If the salary range is an arithmetic sequence with seven terms, determine the raise Mary can expect each year. b) What is her salary in the fifth year? c) What is the first salary in this range that is greater than $30 000? d) What is the total amount that Mary earned in the seven years?

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Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P2–4. Relate arithmetic sequences to linear functions defined over the natural numbers. [CN]	4.1 If three eggs are used for every carrot cake made in a bakery, write a function that determines the number of eggs for n cakes. 4.2 To rent an ice arena, there is an initial charge for cleaning the ice, plus a rental fee for each hour or part of an hour. The rates posted on the board are: Time (h) / Cost ($) Less than 1 / 100 More than 1, less than 2 / 180 More than 2, less than 3 / 260 . . . / Graph the function that models the rates posted on the board.
P2–5. Generate number patterns exhibiting geometric growth. [E, R]	5.1 Insert three numbers between 5 and 80, so that the five numbers form a geometric sequence. 5.2 A store is conducting a Dutch auction. It will take 10% off the cost of an item each day. If an item originally costs $150, find its cost for each of the next five days.

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Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Generalize operations on polynomials to include rational expressions. **Specific Outcomes**	
P1–2. Factor polynomial expressions of the form $ax^2 + bx + c$, and $a^2x^2 - b^2y^2$. [E]	2.1 Factor: a) $5x^2 + 6x - 8$ b) $6x^2 - x - 2$. 2.2 Factor $4x^2 + 20x + 25$. a) Compare the two factors. b) For this special product, what is the relationship between the coefficients of the terms of the factors and the coefficients of the terms of the trinomial? 2.3 Factor $4x^2 - 25$. a) Compare the two factors. b) For this special product, what is the relationship between the coefficients of the terms of the factors and the coefficients of the terms of the binomial? 2.4 For which integral values of k can $4x^2 + kx + 3$ be factored over the set of rational numbers? 2.5 Factor $(x + b)^2 + 6(x + b) + 8$. 2.6 Factor $6x^4 - x^2 - 2$.
P1–3. Find the product of polynomials. [E, R]	3.1 Find the product and simplify: a) $(3x - 4)(2x^2 + 3x + 1)$ b) $(2x - y)^3$.
P1–4. Divide a polynomial by a binomial, and express the result in the forms: • $\frac{P}{D} = Q + \frac{R}{D}$ • $P = DQ + R$ • $P(x) = D(x)Q(x) + R$. [E, R]	4.1 Divide $(3x^3 + 2x^2 - 7x + 8)$ by $(x + 2)$. 4.2 Divide $(t^2 - 3t - 10)$ by $(t - 3)$. 4.3 Divide $(6x^3 - 2x^2 + 7x - 11)$ by $(3x^2 - 2)$. 4.4 When the polynomial $P(t) = 4t^4 - 17t^2 - 36t - 20$ is divided by $(2t - 5)$, the remainder is -60. Express the division in the forms: a) $\frac{P(t)}{2t-5} = Q(t) + \frac{R}{2t-5}$ b) $P(t) = Q(t)(2t - 5) + R$.

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Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P1–5. Determine equivalent forms of simple rational expressions with polynomial numerators, and denominators that are monomials, binomials or trinomials that can be factored. [PS, R]	5.1 Change each rational expression to its simplest equivalent form: a) $\frac{4x^4 - 6x^3 + 2x^2 - 10x}{2x}$ b) $\frac{x^2 - 5x - 6}{x^2 - 36}$ c) $\frac{x^2 + 3x}{x^2 + x - 6}$ d) $\frac{16x^4 - 81y^4}{(4x^2 + 9y^2)^2 (2x^2 - xy - 3y^2)}$.
P1–6. Determine the nonpermissible values for the variable in rational expressions. [C, CN]	6.1 For what value(s) of x are each of the following not defined? Explain your conclusion in each case. a) $\frac{3}{x}$ b) $\frac{-2}{x+1}$ c) $\frac{4}{3x-4}$ d) $\frac{2x+1}{x^2-4}$ e) $\frac{5x}{x^2 - 3x - 4}$ f) $\frac{5x + y}{3x - y}$ g) $\frac{7x^2 - 6xy + 3y^2}{4x^2 - 9y^2}$ h) $\frac{2}{x^3}$ i) $\frac{5}{(x^3 - 1)}$
P1–7. Perform the operations of addition, subtraction, multiplication and division on rational expressions. [E, R]	7.1 For each expression perform the indicated operations, and identify any nonpermissible values. a) $\left(\frac{1}{x}\right) + \left(\frac{3}{2x}\right)$ b) $\left(\frac{4}{x+1}\right) - \left(\frac{1}{x-2}\right)$ c) $\left(\frac{2x+1}{x-1}\right) + \left(\frac{x-1}{x^2 - x - 2}\right)$ d) $\left(\frac{x^2 + 2x + 1}{x-5}\right)\left(\frac{x^2 - 25}{x^2 + 6x + 5}\right)$ e) $\left(\frac{3x^2 + 10x + 3}{x^2 - 9}\right) \div \left(\frac{3x+1}{x-3}\right)$

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Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C] Communication [PS] Problem Solving
[CN] Connections [R] Reasoning
[E] Estimation and Mental Mathematics [T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	f) $\dfrac{3}{\left(\dfrac{2}{x}\right)}$ g) $\dfrac{\left(\dfrac{2x+6}{x+1}\right)}{\left(\dfrac{x+3}{x^2-1}\right)}$ h) $\dfrac{\left(\dfrac{1}{x}+3\right)}{\left(\dfrac{1}{x}-3\right)}$
P1–8. Find and verify the solutions of rational equations. [CN, PS]	8.1 Solve for x, checking for any nonpermissible values. a) $\dfrac{2}{x}=-3$ b) $\dfrac{4}{x}+\dfrac{3}{2x}=\dfrac{11}{4}$ c) $\dfrac{5}{x-1}-\dfrac{2}{x+1}=2$ d) $\dfrac{2x+1}{x+3}-\dfrac{x-2}{x+1}=5$ e) $\dfrac{3}{x^2-25}+\dfrac{2}{x+5}=\dfrac{4}{x-5}$ f) $\dfrac{4}{x-5}+6=\dfrac{4}{x-5}$ 8.2 The average speed of an airplane is five times as fast as the average speed of a passenger train. To travel 400 km, the train requires 4 hours more than the airplane. Find the average speeds of the train and the airplane.

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Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Make and analyze decisions, using expected gains and losses, based on the probabilities of simple events. **Specific Outcomes** P2–6. Connect probabilities to calculated expected gains or losses. [CN, PS, R, V]	6.1 A business person is preparing a proposal for a computer contract worth $12 000. This person estimates that it would cost $1500 to prepare the proposal, and the probability of receiving the contract is estimated to be 0.20. Find this business person's expected gain. 6.2 The Khan family is considering moving from Calgary to Hamilton. In Calgary, Ali earns $46 000 and Kareema earns $34 000. Based on the family's research, if they move, Ali has an estimated probability of 0.85 of finding a job that pays $53 000, and an estimated probability of 0.12 of finding a job that pays $33 000. Otherwise he would be unemployed, receiving $17 000. Kareema has an estimated probability of 0.65 of finding a job that pays $62 000, and an estimated probability of 0.12 of finding a job that pays $33 000. Otherwise she would be unemployed, receiving $11 000. What is the expected gain in salary, if the Khans move to Hamilton? 6.3 Sherry takes a 100-item multiple-choice examination. Each item has four possible choices. She knows 68 of the answers and guesses randomly at the other 32. Calculate her expected number of correct answers.

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Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C] Communication
[CN] Connections
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[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P2–7. Solve decision-making problems involving expected values, and communicate the solutions. [C, PS, R]	7.1 Dave and Tony are playing toss-up with two coins. Dave wins one point, if both coins are heads or both are tails. Tony wins one point, if the two coins are different. After 100 tosses, what are the two players' expected scores? Is this a fair game? 7.2 Joe paid $5 to throw a pair of dice. He wins the sum of the numbers appearing on the top faces of the dice, unless a six appears on either die; then he wins nothing. a) Is this a fair game? b) What difference would it make if the six were changed to a one? c) Justify your answers by analyzing the sample space for this dice throw. 7.3 Obtain collision damage figures for inexperienced drivers and for experienced drivers from an insurance company, and then calculate a fair insurance premium for $1 000 000 liability, $250 deductible collision and $100 deductible comprehensive theft/glass coverage. Do the calculation twice, once for each type of driver. What change in premium would be fair, if the deductible for collision were raised to $1000? 7.4 At what point is it worth it to drop collision coverage on an older vehicle? Show a strategy, and explain the supporting calculations. 7.5 Explain why it is reasonable to insure a house against fire damage, where the probability of collecting is 0.003, but it is not reasonable for a bank, using current interest rates, to make a loan that has a 90% probability of getting repaid. 7.6 The growing of grapes for *Eiswein* involves harvesting the grapes as late as possible in October. As each day passes, the grapes become more valuable, but there is a greater risk of a frost killing the grapes and reducing their value. For a particular year, the value of the grape juice is $2.00/L on October 1, and the value of the juice increases by $0.15/L per day for every day in October. The probability of a killer frost is 0.03 for any particular day in October. After a killer frost, the value of the juice is $1.50/L. On what day does the risk of frost damage outweigh the gain from extra maturing time?

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Strand: Number (Number Operations)
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[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems. **Specific Outcomes** P1–1. Explain and apply the exponent laws for powers of numbers and for variables with rational exponents. [C, E]	1.1 Find the exact value of $\left(\frac{8}{27}\right)^{\left(-\frac{2}{3}\right)}$. 1.2 Write the number expression $7^{\left(\frac{2}{3}\right)}$, using radicals. 1.3 Simplify $\left(\sqrt[5]{x^3}\right)\left(\sqrt[3]{x^2}\right)$. 1.4 Show $\left(\sqrt[3]{-8}\right)x = -2x$. 1.5 Write an equivalent expression for $\sqrt[3]{\sqrt[2]{3x^5}}$, using exponents. 1.6 Prove that $\sqrt{2}$ is an irrational number. 1.7 The 5×5 geoboard shown in the diagram can be used to construct squares whose areas are whole numbers. The sides of the squares can be constructed by joining dots horizontally, vertically or diagonally. What whole number areas can be constructed? Justify your answers with appropriate drawings and calculations. . . . .

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[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P2–1. Perform operations on irrational numbers of monomial and binomial form, using exact values. [E]	1.1 Show that $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$. 1.2 Find an equivalent form of $\left(\frac{3}{\sqrt{5} - \sqrt{2}}\right)$ that has a whole number as its denominator. 1.3 Arrange the following in order from least to greatest: $7,\ 2\sqrt{13},\ 3\sqrt{6},\ 4\sqrt{5},\ 5\sqrt{2}$. Do not use decimal approximations. 1.4 Find the exact value of $\sqrt[3]{128} + 4\left(\sqrt[3]{16}\right)$. 1.5 Find an equivalent form of $\left(3\sqrt{5} + 4\sqrt{2}\right)\left(4\sqrt{5} - 3\sqrt{2}\right)$. 1.6 An equilateral triangle is inscribed in a circle. If the area of the circle is 36π, find the exact area of the equilateral triangle.

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Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Generate and analyze number patterns. **Specific Outcomes**	
P2–2. Generate number patterns exhibiting arithmetic growth. [E, R]	2.1 The first modern Olympiad was held in 1896. Every four years after this date the summer Olympics were held. Given such a framework, reveal what should have been the next five summer Olympic years after 1896. Explain why this pattern was never achieved. 2.2 The output of a northern gold mine has remained constant at 2200 ounces per year. If, at the end of last year, the total output of the mine was 122 600 ounces of gold, what will be the total output at the end of this year? At the end of next year? 2.3 A salesperson receives a base salary of $12 000 per year, plus $100 for every unit sold. What is the salary, if 50 units are sold? 51 units? 52 units? 2.4 For the arithmetic sequence 16, 23, 30, 37, . . . , find the next three terms. 2.5 A pile of bricks is arranged in rows. The numbers of bricks in the rows form an arithmetic sequence. There are 45 bricks in the 5th row and 33 bricks in the 11th row. a) How many bricks are in the first row? b) Write the general term for the sequence. c) What is the maximum number of rows of bricks possible?
P2–3. Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems. [CN, PS, R, T]	3.1 For the arithmetic sequence 7, 11, 15, 19, . . . , find the 29th term. 3.2 Find the sum of the arithmetic series $3 + 7 + 11 + \ldots + 483$. 3.3 Mary's annual salary is on a range from $26 785 in the first year to $34 825 in the seventh year. a) If the salary range is an arithmetic sequence with seven terms, determine the raise Mary can expect each year. b) What is her salary in the fifth year? c) What is the first salary in this range that is greater than $30 000? d) What is the total amount that Mary earned in the seven years?

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Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

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[CN] Connections
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[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P2–4. Relate arithmetic sequences to linear functions defined over the natural numbers. [CN]	4.1 If three eggs are used for every carrot cake made in a bakery, write a function that determines the number of eggs for n cakes. 4.2 To rent an ice arena, there is an initial charge for cleaning the ice, plus a rental fee for each hour or part of an hour. The rates posted on the board are: Time (h) / Cost ($) Less than 1 / 100 More than 1, less than 2 / 180 More than 2, less than 3 / 260 . . . / Graph the function that models the rates posted on the board.
P2–5. Generate number patterns exhibiting geometric growth. [E, R]	5.1 Insert three numbers between 5 and 80, so that the five numbers form a geometric sequence. 5.2 A store is conducting a Dutch auction. It will take 10% off the cost of an item each day. If an item originally costs $150, find its cost for each of the next five days.

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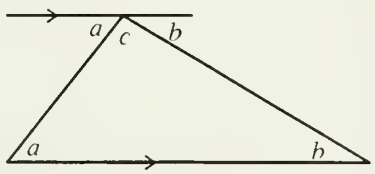
Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

[C] Communication [PS] Problem Solving
[CN] Connections [R] Reasoning
[E] Estimation and Mental Mathematics [T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Apply the principles of mathematical reasoning to solve problems and to justify solutions. **Specific Outcomes**	
P5–1. Differentiate between inductive and deductive reasoning. [CN, R]	1.1 Find, inductively, the sum of the angles of a triangle, by: a) constructing triangles and tearing the corners off b) putting the torn corners together to form a straight line. 1.2 Show, deductively, that the sum of the measures a, b and c is 180°, by: a) drawing a triangle b) using one side as a base and drawing a parallel line segment on the opposite vertex c) knowing that $a = a$, $b = b$, and c is included in both; $\therefore a + c + b = 180°$. 
P5–2. Explain and apply connecting words, such as "and", "or" and "not", to solve problems. [C, PS, R, V]	2.1 Each member of a sports club plays at least one of the following sports: soccer, rugby or tennis. The following information is given: a) 163 play tennis; 36 play tennis and rugby; 13 play tennis and soccer b) 6 play all three sports; 11 play soccer and rugby; 208 play rugby or tennis c) 98 play soccer or rugby. Use this information to determine the number of members in the club. 2.2 On a number line, indicate the location of the sets corresponding to the following: a) $x < 2$ or $x > 5$ b) $x < 2$ and $x > 5$ c) $x < 5$ or $x > 2$ d) $x < 5$ and not $x > 2$.

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Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	2.3 The phrase "*A* or *B*" can be used in ordinary speech in inclusive and exclusive senses, depending on whether "*A* and *B*" is included or excluded. a) Give a practical example of each sense of "*A* or *B*". b) Show the relationship between the inclusive and the exclusive sense of "*A* or *B*" on appropriate Venn diagrams. c) Mathematicians and logicians use the inclusive sense of "*A* or *B*". Justify this choice.
P5–3. Use examples and counterexamples to analyze conjectures. [CN, R]	3.1 Rajiv concluded that whenever he added two prime numbers the sum was always even. Find a counterexample to prove that Rajiv's conjecture is false. 3.2 A science text states that water boils at 100°C. Find a counterexample. 3.3 Mary used her graphing calculator to graph $y = x^x$. She found the screen to be blank for $x < 0$ and made a conjecture that x^x is undefined when $x < 0$. Find an example to show that Mary's conjecture is reasonable. Find a counterexample to show that Mary's conjecture is false. 3.4 The functions $f(x) = \frac{x^2 - 49}{x - 7}$ and $g(x) = x + 7$ are closely related. a) Explain the similarities and the differences between $f(x)$ and $g(x)$. b) How do the graphs of $f(x)$ and $g(x)$ differ from one another?
P5–4. Distinguish between an "if–then" proposition, its converse and its contrapositive. [CN, R]	4.1 Change the statement "Multiples of 3 are always multiples of 6" into "if–then" form, and write the converse and contrapositive of the "if–then" statement. Decide on the truth of all three propositions. 4.2 Create a true proposition whose converse and contrapositive are both true.

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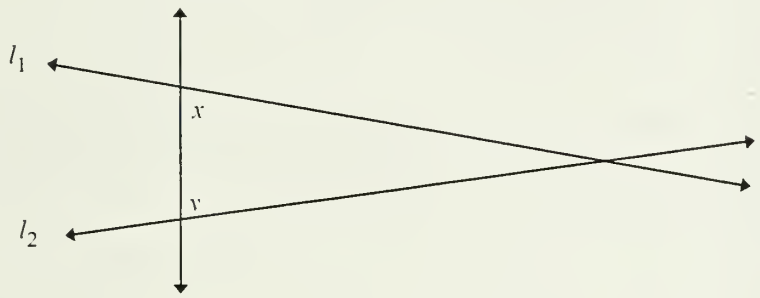
Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P5–5. Prove assertions in a variety of settings, using direct and indirect reasoning. [R]	5.1 Angle ABC is obtuse, and AD is the median of BC. If AD is not an altitude, prove that ABC is a scalene triangle. 5.2 Prove that the medians of a triangle cannot bisect each other. 5.3 In the diagram below, show: a) $x + y < 180°$ b) if $x + y = 180°$, lines l_1 and l_2 are parallel.  5.4 Prove that the difference of squares of two odd numbers is always divisible by 4.

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Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Generalize operations on polynomials to include rational expressions. **Specific Outcomes**	
P1–2. Factor polynomial expressions of the form $ax^2 + bx + c$, and $a^2x^2 - b^2y^2$. [E]	2.1 Factor: a) $5x^2 + 6x - 8$ b) $6x^2 - x - 2$. 2.2 Factor $4x^2 + 20x + 25$. a) Compare the two factors. b) For this special product, what is the relationship between the coefficients of the terms of the factors and the coefficients of the terms of the trinomial? 2.3 Factor $4x^2 - 25$. a) Compare the two factors. b) For this special product, what is the relationship between the coefficients of the terms of the factors and the coefficients of the terms of the binomial? 2.4 For which integral values of k can $4x^2 + kx + 3$ be factored over the set of rational numbers? 2.5 Factor $(x + b)^2 + 6(x + b) + 8$. 2.6 Factor $6x^4 - x^2 - 2$.
P1–3. Find the product of polynomials. [E, R]	3.1 Find the product and simplify: a) $(3x - 4)(2x^2 + 3x + 1)$ b) $(2x - y)^3$.
P1–4. Divide a polynomial by a binomial, and express the result in the forms: • $\frac{P}{D} = Q + \frac{R}{D}$ • $P = DQ + R$ • $P(x) = D(x)Q(x) + R$. [E, R]	4.1 Divide $(3x^3 + 2x^2 - 7x + 8)$ by $(x + 2)$. 4.2 Divide $(t^2 - 3t - 10)$ by $(t - 3)$. 4.3 Divide $(6x^3 - 2x^2 + 7x - 11)$ by $(3x^2 - 2)$. 4.4 When the polynomial $P(t) = 4t^4 - 17t^2 - 36t - 20$ is divided by $(2t - 5)$, the remainder is -60. Express the division in the forms: a) $\frac{P(t)}{2t-5} = Q(t) + \frac{R}{2t-5}$ b) $P(t) = Q(t)(2t - 5) + R$.

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Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P1–5. Determine equivalent forms of simple rational expressions with polynomial numerators, and denominators that are monomials, binomials or trinomials that can be factored. [PS, R]	5.1 Change each rational expression to its simplest equivalent form: a) $\frac{4x^4 - 6x^3 + 2x^2 - 10x}{2x}$ b) $\frac{x^2 - 5x - 6}{x^2 - 36}$ c) $\frac{x^2 + 3x}{x^2 + x - 6}$ d) $\frac{16x^4 - 81y^4}{(4x^2 + 9y^2)^2 (2x^2 - xy - 3y^2)}$.
P1–6. Determine the nonpermissible values for the variable in rational expressions. [C, CN]	6.1 For what value(s) of x are each of the following not defined? Explain your conclusion in each case. a) $\frac{3}{x}$ b) $\frac{-2}{x+1}$ c) $\frac{4}{3x-4}$ d) $\frac{2x+1}{x^2-4}$ e) $\frac{5x}{x^2 - 3x - 4}$ f) $\frac{5x + y}{3x - y}$ g) $\frac{7x^2 - 6xy + 3y^2}{4x^2 - 9y^2}$ h) $\frac{2}{x^3}$ i) $\frac{5}{(x^3 - 1)}$
P1–7. Perform the operations of addition, subtraction, multiplication and division on rational expressions. [E, R]	7.1 For each expression perform the indicated operations, and identify any nonpermissible values. a) $\left(\frac{1}{x}\right) + \left(\frac{3}{2x}\right)$ b) $\left(\frac{4}{x+1}\right) - \left(\frac{1}{x-2}\right)$ c) $\left(\frac{2x+1}{x-1}\right) + \left(\frac{x-1}{x^2 - x - 2}\right)$ d) $\left(\frac{x^2 + 2x + 1}{x-5}\right)\left(\frac{x^2 - 25}{x^2 + 6x + 5}\right)$ e) $\left(\frac{3x^2 + 10x + 3}{x^2 - 9}\right) \div \left(\frac{3x+1}{x-3}\right)$

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Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C] Communication [CN] Connections [E] Estimation and Mental Mathematics [PS] Problem Solving [R] Reasoning [T] Technology [V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	f) $\dfrac{3}{\left(\dfrac{2}{x}\right)}$ g) $\dfrac{\left(\dfrac{2x+6}{x+1}\right)}{\left(\dfrac{x+3}{x^2-1}\right)}$ h) $\dfrac{\left(\dfrac{1}{x}+3\right)}{\left(\dfrac{1}{x}-3\right)}$
P1–8. Find and verify the solutions of rational equations. [CN, PS]	8.1 Solve for x, checking for any nonpermissible values. a) $\dfrac{2}{x}=-3$ b) $\dfrac{4}{x}+\dfrac{3}{2x}=\dfrac{11}{4}$ c) $\dfrac{5}{x-1}-\dfrac{2}{x+1}=2$ d) $\dfrac{2x+1}{x+3}-\dfrac{x-2}{x+1}=5$ e) $\dfrac{3}{x^2-25}+\dfrac{2}{x+5}=\dfrac{4}{x-5}$ f) $\dfrac{4}{x-5}+6=\dfrac{4}{x-5}$ 8.2 The average speed of an airplane is five times as fast as the average speed of a passenger train. To travel 400 km, the train requires 4 hours more than the airplane. Find the average speeds of the train and the airplane.

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Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Represent and analyze situations that involve expressions, equations and inequalities. **Specific Outcomes**	
P3–1. Solve nonlinear equations: • by factoring • graphically. [CN, T, V]	1.1 Solve by factoring: a) $x^2 - 2x = 24$ b) $x^3 = 1$ c) $2x^2 + 9x - 5 = 0$ d) $7x^2 + 4x - 11 = 0$. 1.2 Solve each of the above graphically. For example, $x^2 - 2x = 24$ can be solved by graphing $y = x^2 - 2x$ and $y = 24$ and using the points of intersection to determine the solution. 1.3 Solve $3x^2 + 1 = 10x - 2$ graphically in two different ways. Is there one way that gives more reliable results? Explain your procedures and the results obtained.
P3–2. Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials. [E, PS, T]	2.1 The polynomial $P(x) = 4x^3 + bx^2 + cx + 11$ has a remainder of -7 when divided by $(x + 2)$ and a remainder of 14 when divided by $(x - 1)$. Find the values of b and c. 2.2 Factor $x^3 - 2x^2 - 5x + 6$.
P3–3. Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V]	3.1 Find the solutions to the following system: $y = x^2$ $y = 8 - x^2$. 3.2 Graphically, find the solution set to the following system: $y = 3x + 2$ and $y = 2^x$. How do you know that the solution set is complete?

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Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	3.3 The world's population grows by 2% per year. The world food production can sustain an additional 200 million people per year. In 1987 the population was 5 billion, and food production could sustain 6 billion people. The population growth can be modelled by the equation $P_1 = 5(1.02)^n$, with the food production being modelled by $P_2 = 0.2n + 6$. The variable n is the number of years after 1987. a) When does $P_1 = P_2$? b) If $P_1 > P_2$ is true, when does this happen, and how is this inequality interpreted?
P3–4. Solve systems of linear equations, in three variables: • algebraically • with technology. [CN, PS, T, V]	4.1 Determine the solution to the following system: $2x + y - z = 3$ $x + 2y + z = 0$ $3x - y - 2z = 11$. 4.2 The total revenue R is a quadratic function of the price p of books sold. So $R = ap^2 + bp + c$. Find the values of a, b and c, if the revenue is \$6000 at a price of \$30, \$6000 at a price of \$40 and \$5000 at a price of \$50.

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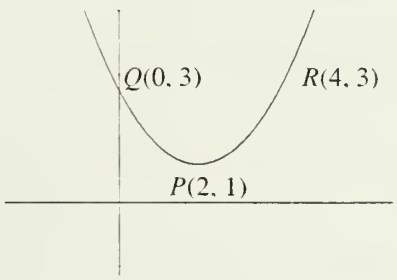
Strand: Patterns and Relations (Relations and Functions)
Students will:
- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Examine the nature of relations with an emphasis on functions. **Specific Outcomes**	
P4–1. Perform operations on functions and compositions of functions. [CN, E, PS]	1.1 If $f(x) = 3x + 2$ and $g(x) = x^2$, find: a) $3f(x)$ b) $f(x) \bullet g(x)$ c) $f(x) + g(x)$ d) $f(g(x))$ e) $f(f(x))$. 1.2 A ball thrown in the air has a velocity given by $v(t) = 49 - 9.8t$. The kinetic energy function $K(v)$ is given by $K(v) = 0.4v^2$. Express the ball's kinetic energy as a function $K(t)$ of time.
P4–2. Determine the inverse of a function. [CN, R, V]	2.1 Graph the inverse of $y = \frac{x}{(x-1)}$, and determine the equation, domain and range of the inverse. 2.2 Sketch the inverse of the following.  2.3 Sketch the inverse of $f(x) = x^2$. 2.4 If $f(x) = 2x - 1$ and $g(x) = \frac{x+1}{2}$, find $f(g(x))$ and $g(f(x))$, and show that the functions $f(x)$ and $g(x)$ are inverses of each other. 2.5 Determine the domain and range for each of the functions in illustrative examples 2.2 and 2.3. 2.6 Is the inverse of $f(x) = 2x - 5$ a function?

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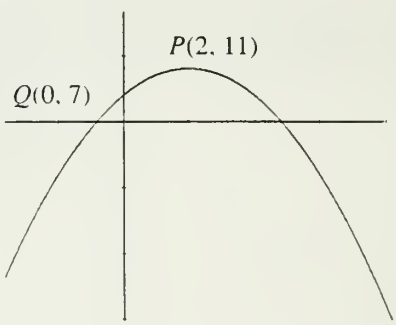
Strand: Patterns and Relations (Relations and Functions)
Students will:
- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate. **Specific Outcomes**	
P4–3. Connect algebraic and graphical transformations of quadratic functions, using completing the square as required. [CN, T, V]	3.1 Graph $f(x) = 2x^2 + 5x - 7$. 3.2 Give a list of events or situations that might be described by a quadratic, parabolic, shape. 3.3 Given the graph of $y = x^2$, sketch $y = -2(x - 3)^2 - 4$. 3.4 Given the graph of $y = x^2$, what is the equation for the transformed graph shown here?  3.5 Rewrite the equation of $f(x) = 2x^2 - 12x + 13$ in the form $f(x) = a(x - p)^2 + q$, and graph the function.
P4–4. Model real-world situations, using quadratic functions. [CN, PS]	4.1 Computer software programs are sold to students for $20 each, and 300 students are willing to buy them at that price. For every $5 increase in price, there are 30 fewer students willing to buy the software. What is the maximum revenue? 4.2 What is the maximum rectangular area that can be enclosed by 120 m of fencing, if one of the sides of the rectangle is an existing wall?

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Strand: Patterns and Relations (Relations and Functions)
Students will:
- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P4–5. Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using: • factoring • the quadratic formula • graphing. [CN, E, T, V]	5.1 Solve $3x^2 - 5x + 2 = 0$ algebraically and by graphing the corresponding function $f(x) = 3x^2 - 5x + 2$. 5.2 When bicycles are sold for \$280 each, a cycle store can sell 80 in a season. For every \$10 increase in the price, the number sold drops by 3. a) Represent the sales revenue as a quadratic function of either the number sold or the price. b) What is the number sold, and the price, if the total sales revenue is exactly \$20 000? c) What is the range of prices that will give a sales revenue that exceeds \$15 000? 5.3 Write a quadratic equation whose roots are $\frac{3}{2}$ and $-\frac{1}{4}$. Is this equation unique?
P4–6. Determine the character of the real and non-real roots of a quadratic equation, using: • the discriminant in the quadratic formula • graphing. [C, R, T, V]	6.1 If $3x^2 - mx + 2 = 0$ can be factored, what values of m are possible? 6.2 Discuss the implications of a negative discriminant when describing the zeros of a quadratic function. 6.3 Given $3x^2 - mx + 3 = 0$: a) For what value(s) of m would one root be double the other? b) For what values of m would the roots not be real? 6.4 The profit y for publishing a book is given by the equation $y = -5x^2 + 400x - 3000$, where x is the selling price per book. a) Is it possible to set a selling price that will earn a total profit of \$6000? Explain your solution with reference to appropriate equations and graphs. b) What range of selling prices allows the publisher to make a profit on this book?

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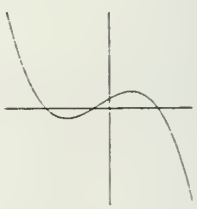
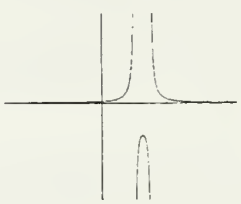
Strand: Patterns and Relations (Relations and Functions)
Students will:
- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P4–7. Describe, graph and analyze polynomial and rational functions, using technology. [C, R, T, V]	7.1 Determine if each of the following examples is a rational function, a polynomial function or some other type of function, and justify your conclusion. a) $y = x^2 - 3x + \sqrt{7}$ b) $y = (x-5)^{-1}$ c) $y = \frac{1}{5}x^4 + 3x^3 - 12x - 0.75$ d) $y = \sqrt{7x^5} + x^2$ e) $y = 2^x - 9$ f) $y = \frac{3x-7}{x^2 - 5x + 6}$ 7.2 Examine the following graphs. Which could be graphs of rational functions, and which could be graphs of polynomial functions? a) b)  c)  d) 7.3 Graph $y = x^2(x^2 - 4)$. What is the domain and range of this function? 7.4 Graph $y = x^2 - 1$, identify the zeros of this function, and use these to predict the asymptotes of $y = \frac{1}{(x^2 - 1)}$. Then graph $y = \frac{1}{(x^2 - 1)}$, using a graphing tool. Compare the two graphs, considering domain, range, asymptotes and zeros.

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Strand: Patterns and Relations (Relations and Functions)
Students will:
- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	7.5 Use a graphing tool to graph $y = \frac{x^2}{(x^2 - 4)}$ and to predict the domain, range and zeros. Describe the symmetry. 7.6 Use technology to graph $f(x) = x^3 - 4x^2 + k$ for various values of k. a) Estimate the values of k for which the equation $f(x) = 0$ appears to have a double root. b) Show that $k = 0$ ensures that $f(x) = 0$ has a double root. c) Show that $k = \frac{256}{27}$ ensures that $f(x) = 0$ has a double root.
P4–8. Formulate and apply strategies to solve absolute value equations, radical equations, rational equations and inequalities. [CN, R, V]	8.1 Sketch $f(x) = \lvert x - 1\rvert - 4$, and determine the values of x for which $f(x) > 0$. 8.2 Solve for x: a) $\lvert x - 1\rvert > 7$ b) $\sqrt{(x-1)} + \sqrt{(x+4)} = 5$ c) $\sqrt{(x+2)} > \frac{x}{x+2}$ d) $\lvert x - 1\rvert + \lvert 2x - 1\rvert > 7$.
	8.3 The point P lies on the y-axis, while points A and B are $(-9, 0)$ and $(5, 0)$ respectively. If $PA + PB$ is 28 units long, determine the coordinates of P.

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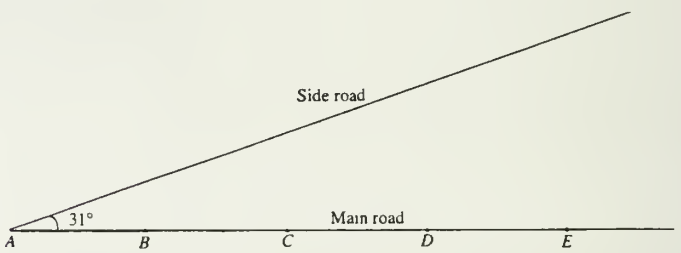
Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Solve problems involving triangles, including those found in 3-D and 2-D applications. **Specific Outcomes** P3–5. Solve problems involving ambiguous case triangles in 3-D and 2-D. [CN, PS, R, T]	5.1 An 11 cm long line AB is drawn at an angle of 44° to a horizontal line AE. A circle with centre B and a radius of 9 cm is drawn, cutting the horizontal line at points C and D. Calculate the length of the chord CD. 5.2 The line segment of equation $y = 2.4x$, passes through $A(0, 0)$ and $C(5, 12)$, has a length of 13 and makes an angle of 67.3° with the horizontal x-axis. a) What points are located with $CB = 10$ and AB horizontal? b) Check your answer by determining the intersection points of the circle $(x - 5)^2 + (y - 12)^2 = 100$ and the line $y = 0$. c) Use a suitable diagram to explain why the answers to a) and b) are the same. 5.3  Streetlights A, B, C, D and E are placed 50 m apart on the main road, as indicated on the diagram. The light from a streetlight can travel 24 m. Determine the furthest point on the side road that is lighted and the length of side road that is illuminated by both streetlight C and streetlight D.

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Strand: Shape and Space (3-D Objects and 2-D Shapes)
Students will:
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Solve coordinate geometry problems involving lines and line segments, and justify the solutions. **Specific Outcomes**	
P3–6. Solve problems involving distances between points and lines. [CN, PS, R]	6.1 Determine the shortest distance from (3, 4) to the line $2x - 5y = 7$. 6.2 The lines $y = 3x + 1$ and $y = 3x - 9$ are parallel. Determine the vertical distance between the two lines, the horizontal distance between the two lines and the shortest distance between the two lines.
P3–7. Verify and prove assertions in plane geometry, using coordinate geometry. [C, R, V]	7.1 Given $A = (-1, 3)$, $B = (0, 5)$ and $C = (-2, 6)$: a) Verify that ABC is a right-angled triangle. b) Is ABC isosceles? Justify your assertion. c) If M is the midpoint of AB and N is the midpoint of AC, prove that MN is parallel to BC. d) Find a point D so that $ABCD$ is a parallelogram. Prove that $ABCD$ is not a rectangle. 7.2 Use coordinate geometry to prove that: a) the diagonals of any parallelogram bisect one another b) if ABC is any triangle, with M as the midpoint of AB and N as the midpoint of AC, then MN is parallel to BC and is half its length. 7.3 Use coordinate geometry to divide the line segment with end points $A(4, 7)$ and $B(-3, 8)$ into five congruent parts.

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Strand: Shape and Space (3-D Objects and 2-D Shapes)
Students will:
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Develop and apply the geometric properties of circles and polygons to solve problems. **Specific Outcomes** P5–6. Prove the following general properties, using established concepts and theorems: • the perpendicular bisector of a chord contains the centre of the circle • the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc (for the case when the centre of the circle is in the interior of the inscribed angle) • the inscribed angles subtended by the same arc are congruent • the angle inscribed in a semicircle is a right angle • the opposite angles of a cyclic quadrilateral are supplementary • a tangent to a circle is perpendicular to the radius at the point of tangency • the tangent segments to a circle from any external point are congruent • the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord • the sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles. [C, R, V]	6.1 a) For what values of c does the line $y = c$ touch the circle $x^2 + y^2 = r^2$? b) Use the result from part a) to show that the tangent to a circle is perpendicular to the radius at the point of tangency. 6.2 Show that the angle inscribed in a semicircle is a right angle. 6.3 The chord AB is one side of a regular polygon of n sides. The polygon is inscribed in a circle. If D is any other vertex of the polygon, prove that the magnitude of angle ADB is $\frac{180°}{n}$.

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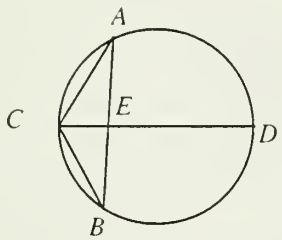
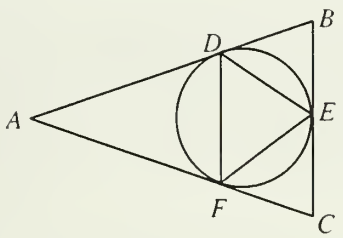
General and Specific Outcomes	Illustrative Examples [Discretionary]
P5–7. Solve problems, using a variety of circle properties, and justify the solution strategy used. [PS, R, V]	7.1 If diameter CD is perpendicular to chord AB at E, prove that triangle ABC is isosceles.  7.2 Determine the measure of $\angle BAC$, if $\angle DEF = 60°$ and $\angle EFC = 70°$. Provide a reason for each step in the solution strategy.  7.3 A chain on a bicycle connects two gear wheels of diameters 9 cm and 19 cm respectively. The centres of the gear wheels are 87 cm apart. Find the minimum length of the chain.

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Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C] Communication
[CN] Connections
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[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Make and analyze decisions, using expected gains and losses, based on the probabilities of simple events. **Specific Outcomes** P2–6. Connect probabilities to calculated expected gains or losses. [CN, PS, R, V]	6.1 A business person is preparing a proposal for a computer contract worth $12 000. This person estimates that it would cost $1500 to prepare the proposal. and the probability of receiving the contract is estimated to be 0.20. Find this business person's expected gain. 6.2 The Khan family is considering moving from Calgary to Hamilton. In Calgary, Ali earns $46 000 and Kareema earns $34 000. Based on the family's research, if they move, Ali has an estimated probability of 0.85 of finding a job that pays $53 000, and an estimated probability of 0.12 of finding a job that pays $33 000. Otherwise he would be unemployed, receiving $17 000. Kareema has an estimated probability of 0.65 of finding a job that pays $62 000, and an estimated probability of 0.12 of finding a job that pays $33 000. Otherwise she would be unemployed, receiving $11 000. What is the expected gain in salary, if the Khans move to Hamilton? 6.3 Sherry takes a 100-item multiple-choice examination. Each item has four possible choices. She knows 68 of the answers and guesses randomly at the other 32. Calculate her expected number of correct answers.

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General and Specific Outcomes	Illustrative Examples [Discretionary]
P2–7. Solve decision-making problems involving expected values, and communicate the solutions. [C, PS, R]	7.1 Dave and Tony are playing toss-up with two coins. Dave wins one point, if both coins are heads or both are tails. Tony wins one point, if the two coins are different. After 100 tosses, what are the two players' expected scores? Is this a fair game? 7.2 Joe paid $5 to throw a pair of dice. He wins the sum of the numbers appearing on the top faces of the dice, unless a six appears on either die; then he wins nothing. a) Is this a fair game? b) What difference would it make if the six were changed to a one? c) Justify your answers by analyzing the sample space for this dice throw. 7.3 Obtain collision damage figures for inexperienced drivers and for experienced drivers from an insurance company, and then calculate a fair insurance premium for $1 000 000 liability, $250 deductible collision and $100 deductible comprehensive theft/glass coverage. Do the calculation twice, once for each type of driver. What change in premium would be fair, if the deductible for collision were raised to $1000? 7.4 At what point is it worth it to drop collision coverage on an older vehicle? Show a strategy, and explain the supporting calculations. 7.5 Explain why it is reasonable to insure a house against fire damage, where the probability of collecting is 0.003, but it is not reasonable for a bank, using current interest rates, to make a loan that has a 90% probability of getting repaid. 7.6 The growing of grapes for *Eiswein* involves harvesting the grapes as late as possible in October. As each day passes, the grapes become more valuable, but there is a greater risk of a frost killing the grapes and reducing their value. For a particular year, the value of the grape juice is $2.00/L on October 1, and the value of the juice increases by $0.15/L per day for every day in October. The probability of a killer frost is 0.03 for any particular day in October. After a killer frost, the value of the juice is $1.50/L. On what day does the risk of frost damage outweigh the gain from extra maturing time?

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REFERENCES

Bye, Marshall P. et al. *Holtmath 11*. Toronto, ON: Holt, Rinehart and Winston of Canada, Limited, 1988. Question 6, page 313. Reprinted with permission.

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